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Sem: IV (Session: 2023-27)

Subject: Electrodynamics

Unit: Maxwell's Equations

Topic: Electrodynamics before Maxwell

Before Maxwell, following four equations were governing the Electromagnetic theory:→

i) Gauss's Law:

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$$

where, $\vec{\nabla}$ → del operator
$\vec{E}$ → Electric field

$\vec{\nabla} \cdot \vec{E} \Rightarrow$ Divergence of $\vec{E}$

$\epsilon_0 \rightarrow$ electric constant or

Permittivity of free space

$\approx 8.854 \times 10^{-12}$ F/m

$\rho \rightarrow$ charge density

ii) $\vec{\nabla} \cdot \vec{B} = 0$

$\vec{B} \rightarrow$ Magnetic field

iii) Faraday's Law :

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

where $\vec{\nabla} \times \vec{E} \rightarrow$ Curl of $\vec{E}$

iv) Ampere's Law :

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

where $\vec{J}$ is the current density

$\rightarrow$

Since divergence of a curl is always zero. ~~This~~ i.e, $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$

This can also be checked with eq^n iii) and iv) of Electromagnetic theory

<u>for eq^n iii)</u>

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$= -\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{B})$$

since, $\vec{\nabla}\cdot\vec{B} = 0$ (eq^n (ii))

$$\Rightarrow \vec{\nabla}\cdot(\vec{\nabla}\times\vec{E}) = 0$$

<u>for Eq^n (iv)</u>

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\cdot(\mu_0\vec{J})$$

$$= \mu_0(\vec{\nabla}\cdot\vec{J})$$

Now,

(i) for steady currents

$$\vec{\nabla}\cdot\vec{J} = 0$$

Hence. $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = 0$ for steady currents.

ii) for Non steady currents :-

$$\vec{\nabla}\cdot\vec{J} \neq 0$$

Now, Using Continuity Equation :-

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$ (for non steady currents)

$$= -\frac{\partial}{\partial t}\left(\epsilon_0\,\vec{\nabla}\cdot\vec{E}\right)$$

↳ eq^n (i)

$$= -\vec{\nabla}\cdot\left(\epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

Now, If we combine $\epsilon_0\frac{\partial\vec{E}}{\partial t}$ with J in Ampere's Law. The total sum with new current density will fix the problem of $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B})$.

i.e, $$\boxed{J' = J + \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

→ Current density for non steady current.

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{B} = \mu_0 \left[J + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right]}$$

$$\Rightarrow \vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \cdot \left[\mu_0 \left\{J + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right\}\right]$$

where $J + \epsilon_0 \frac{\partial \vec{E}}{\partial t} = 0$. $\Rightarrow \boxed{J = -\epsilon_0 \frac{\partial \vec{E}}{\partial t}}$

→ This suggest :→

A changing electric field $\left(\frac{\partial \vec{E}}{\partial t}\right)$ introduces magnetic field (μJ)

The extra term in ~~non~~ current density for the non steady currents reffered as :→

Displacement Current :→

$$\boxed{J = \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$